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A Note on Option Value and the Expected Value of Consumer's Surplus

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PREFACE

Although environmental policy decisions frequently are based on other criteria, cost-benefit analysis plays an important role in the decision-making process. Questions concerning the costs and benefits of alternative courses of action are being explored in several of REN's studies of natural resource and environmental management problems. This paper explores a general methodological problem that has arisen in applications of cost-benefit analysis of environmental resource decisions.

Several investigators claim that the usual methods of estimating the benefits of environment preservation understate systematically those benefits by an amount called "option value". Loosely put, option value is the benefit that potential (but uncertain) users of environmental services derive from avoiding the risk that these services would be unavailable. A proof has been offered that these option value benefits are always non-negative. Other investigators have offered proofs that option value benefits may be either positive or negative and that, on *a priori* grounds, there is no way to determine whether or not usual benefits valuation methods systematically understate benefits of environmental preservation.

This paper attempts to reconcile these contradictory results. Helpful comments and suggestions from Jesse Ausubel, Donald Erlenkotter, and Mark Pauly are acknowledged gratefully. Nono of these kind individuals is to be held responsible for any errors, ambiguities, or other faults that may remain.

CONTENTS

INTRODUCTION

In roughly contemporaneous papers, Schmalensee (1972) (hereafter "S") and Krutilla, Cicchetti, Freeman and Russell (1972) (hereafter "KC") reach apparently conflicting conclusions about the sign taken by option value.* Under the assumption that an individual's utility function is concave in income, S concluded that the sign of "option value depends on the details of individual preferences and circumstances and may be either positive or negative."** In contrast, KC conclude that option value is always non-negative.

The practical significance of these conclusions is enormous. Analysts typically use the expected value of consumer's surplus as a measure of the uncertain future economic benefits of projects. KC's analysis implies that this practice will tend systematically to understate these benefits. S's analysis implies that while the expected value of consumer's surplus is not an exact measure of future benefits, there is no reason to conclude that it is biased systematically either upward or downward.

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*Option value is the difference between the maximum amount that an individual who maximizes expected utility would pay for an option which guaranteed his/her right to consume at a stated set of prices, and the expected value of consumer's surplus from consuming at those prices. More rigorous definitions of option value are given in Section 2 of this paper.

**Schmalensee 1972:814.

Although the subsequent literature on option value reaffirms S's general conclusion, nowhere to my knowledge has an explicit explanation been offered for the contradictory result obtained by KC. Bishop and Cicchetti (1975) hint that: "By separating the problem the way he [i.e., Schmalensee] does, the uncertainty and trade-offs of the early literature are lost."* Bohm (1975) notes that in the special case employed in KC's analysis (i.e., a two state world in which the probability of each state is 0.5 and in which consumer's surplus associated with one of the states is 0), option value is positive if the marginal utility of income in the state in which consumer surplus is zero. Bohm and S also note many other specific cases in which option value is positive. However, neither explicitly diagnoses or states the source of KC's more definite and seemingly contradictory result.

What, then, led KC to reach the conclusion they did? The question is an important one. If it could be shown, as Bishop and Cicchetti allege, that KC's formulation better reflects actual conditions of choice under uncertainty, then their conclusions concerning option value would have to be taken seriously.

The purpose of this note is to show exactly why KC reach a different conclusion than does S concerning the sign of option value. The reason has nothing to do with the particular values of probabilities of states of nature, the number of states, the values assumed for consumer surplus in any of the states, or the relative values of marginal utilities of income. I will show that KC's conflicting conclusions stem from two assumptions (one of which is not made explicit) that in all cases impose stronger conditions on individual "preferences and circumstances" than are imposed by S.

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*Bishop and Cicchetti 1975:111. Footnote 4.

NOTATION, DEFINITIONS, AND ASSUMPTIONS

To facilitate comparison of the S and KC formulations and results, let us adopt the following notation, definitions, and basic assumptions:

n = number of states

π_i = probability that state i occurs

U^i = indirect utility function conditional on the occurrence of state i, which is assumed to be twice continuously differentiable and strictly concave

P^* = vector of prices

P = alternative vector of prices such that $P \geqq P^*$

Y_i = income, conditional on the occurrence of the i^{th} state

OPC = option price, the amount that a consumer would be willing to pay to secure price vector P* instead of price vector P in the future, i.e.,

$$\sum_{i=1}^{n} \pi_i U^i(P^*, Y_i - OPC) = \sum_{i=1}^{n} \pi_i U^i(P, Y_i)$$

SC_i = consumer's surplus in state i when prices are P* instead of P; i.e., $U^i(P^*, Y_i - SC_i) = U^i(P, Y_i)$

OV = option value =

$$OPC - \sum_{i=1}^{n} \pi_i SC_i$$

The definitions and assumptions spelled out above are adopted by both KC and S. In addition to these basic assumptions, KC adopt two additional assumptions. These are best explained by reference to a diagram. In Figure 1, indifference curves from two preference maps, corresponding to two different states of nature, are drawn. The curves labeled U_1^1 and U_2^1 represent indifference curves from the individual's preference map if state 1 occurs (the subscripts index different utility levels) and those labeled U_1^2 and U_2^2 represent different utility levels in state 2. The two goods whose quantities are shown on the axes of Figure 1 are "Hicks-Marshall" money (denoted by Y) and some other good X. The budget line Y_0A is drawn assuming that the preferred price vector, P*, obtains.

As can be seen in the diagram, if state 2 occurs, the quantity demanded of X is 0 and the resulting level of utility is U_1^2; if state 1 occurs, the quantity demanded of X is X_1 and the resulting level of utility is U_1^1.

The first additional assumption made by KC provides a means for relating the levels of utility in different states of nature. This assumption, as explained by KC, is as follows:

> For any given level of income (say Y_0 in Figure 1), if the individual did not demand good X, he/she would choose a point on the Y axis and experience a certain level of utility (U_1^2 in Figure 1); if he/she were to demand the good (assuming that it is available), he/she would choose a tangency point on the budget line associated with income Y_0 and experience a given level of utility (U_1^1 in Figure 1). Assume that the alternative outcomes have the same utility. That is, $U_1^2 = U_1^1$.*

Put succinctly, KC's assumption is that if income is identical in all states, utilities in all states of nature are equal at the preferred price vector. In terms of the notation set forth above, this assumption requires that

$$U^i(P^*,Y_i) = U^j(P^*,Y_j) \quad \text{if } Y_i=Y_j$$

for all states of nature i and j.

The second additional assumption made by KC is nowhere explicitly stated. Rather it is implicit in the derivation of their conclusion. It arises in their construction of the relationship between utility and income, and their use of this constructed relationship to derive results on the sign of option value. KC's assumption is that income is the same in all states of nature.**

*See KC 1972:103. I have paraphrased their statement slightly to accomodate the notation I have adopted.

**See KC 1972:104-105.

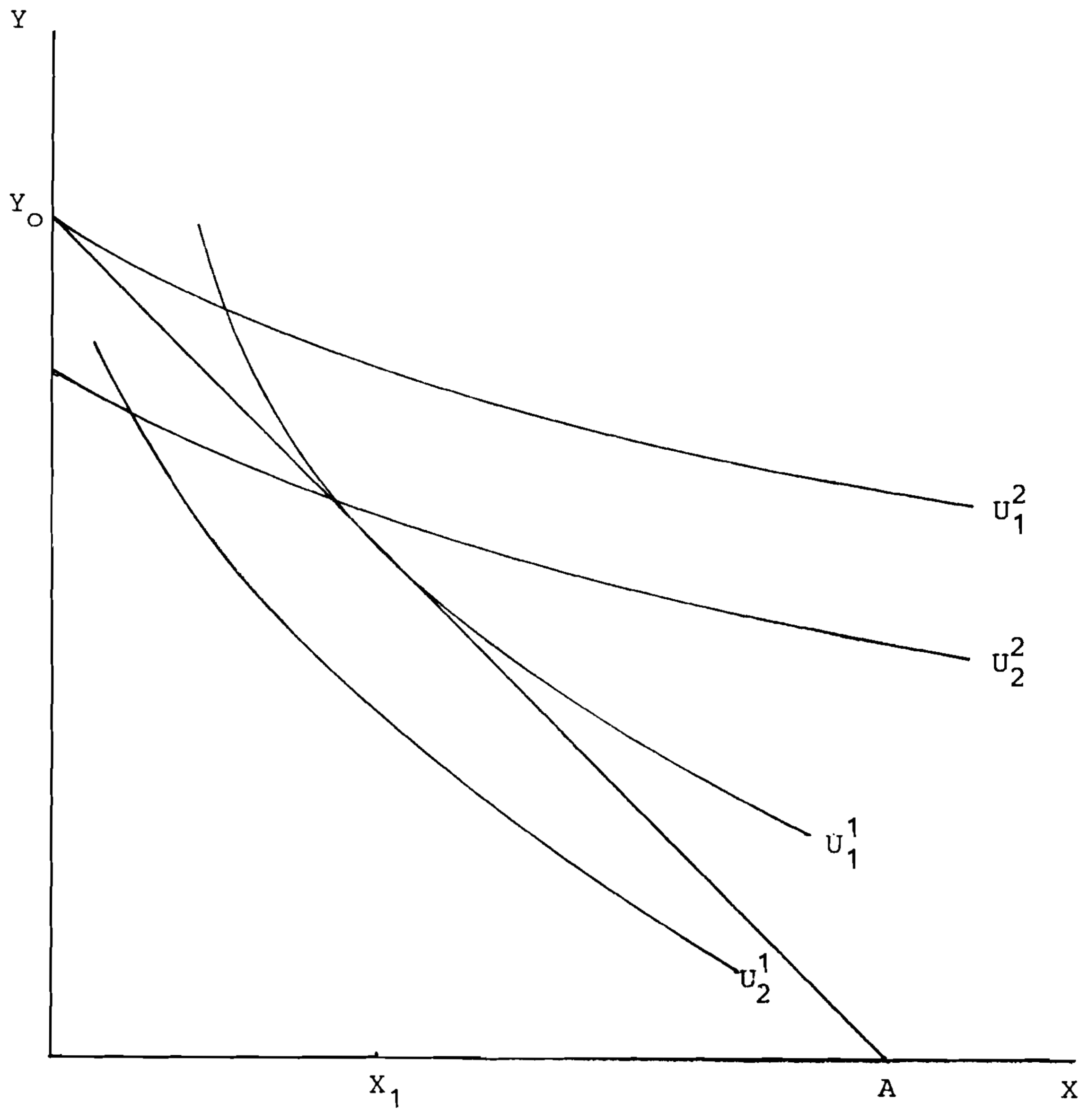

Figure 1. Relationship between utilities on different preference maps.

DERIVATIONS

It is relatively easy to show how KC, given all of these assumptions (i.e., the basic assumptions and the two additional assumptions), reach the conclusion that option value is non-negative. To do this, let us first relate our mathematical definition of option price to our mathematical definition of consumer's surplus (see the definitions of option price and consumer's surplus in Section 2). Carrying out the required substitution, we obtain

$$\sum_{i=1}^{n} \pi_i U^i(P^*, Y_i - OPC) = \sum_{i=1}^{n} \pi_i U^i(P^*, Y_i - SC_i) \qquad (1)$$

Approximating $U^i(P^*, Y_i - SC_i)$ by expanding to the first-order about Y_i-OPC, and making use of the fact that the U^i are concave (by assumption), we obtain

$$U^i(P^*, Y_i - SC_i) \leq U^i(P^*, Y_i - OPC) + U^i_y(P^*, Y_i - OPC)(OPC - SC_i),$$

where U^i_y is the derivative of U^i with respect to Y_i, the so-called "marginal utility of income." Substituting this relationship into equation (1) and collecting terms, we obtain

$$\sum_{i=1}^{n} \pi_i U^i_y(P^*, Y_i - OPC)(OPC - SC_i) \geq 0 \qquad (2)$$

Equation (2) is (one of) the fundamental equations obtained by S in his analysis of option value. He demonstrates that, making use only of the basic assumptions spelled out in Section 2, one cannot conclude from this relationship whether option value is positive or negative.

Now let us impose the two additional assumptions made by KC. Taken jointly, these assumptions imply that the utilities in all states of nature are equal. It follows that the derivatives of utilities in all states of nature with respect to income are equal, i.e., that $U^i_y = U^j_y$ for all i and j. Inspection of equation (2) reveals immediately the conclusion that follows in this case. Option value is always non-negative under these circumstances.

CONCLUSIONS

It is clear that the additional assumptions made by KC are stronger than most analysts would want to adopt. In effect, the option offered in the KC analysis guarantees the purchaser that a specified utility level will be obtained in all future states. Very few practical investment options are of this sort. Under the more general formulations adopted by S, purchase of an option entitles its holder only to a given expected utility level. KC's conclusion, although impeccably correct given the assumptions under which it was derived, is therefore of limited practical significance.

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